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TECHNICAL NOTE

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A METHOD FOR DETERMINING THE PROPELLANT REQUIRED TO MAINTAIN AN EARTH ORBITAL VEHICLE IN A USEFUL ORBIT BETWEEN TWO SPECIFIED ALTITUDE LIMITS

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SUMMARY

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A set of working charts is presented which allows rapid determination of the propellant requirements for maintaining an earth orbital vehicle in a permanent circular orbit in the altitude range of 160 to 300 nautical miles. The orbit is allowed to decay to a predetermined altitude below the original orbital altitude, and a Hohmann transfer maneuver is used to return the vehicle to a circular orbit at its original altitude. The number of such corrections required per year, and the velocity requirement to make a correction are combined to give the yearly propellant consumption requirement.

Two atmosphere models were used, the 1959 ARDC model atmosphere, and the 1962 U.S. standard atmosphere.

The results indicate that at the higher altitudes propellant mass fractions of the order of 1 percent are required to sustain the orbit if the orbit is permitted to decay only a small percentage of the original orbit altitude before a correction is made. At altitudes below 180 nautical miles, the propellant consumption required to maintain the orbit for a year exceeds 10 percent of the total station weight.

INTRODUCTION

With the recent discovery of a low-altitude, manmade radiation belt in the vicinity of Brazil (ref. 1), it might be required to place permanent manned space stations, with moderate orbital inclinations, into lower altitude orbits than might be desired from orbital lifetime considerations in order to keep the radiation shielding requirements for the space station to a minimum. Since circular orbits in the altitude range of 100 nautical miles to 300 nautical miles still encounter enough drag to cause a considerable altitude decay over long periods of time, some means of propulsion must be provided in the space station to permit the station to maintain a useful orbit for the required length of time. The analysis of this paper yields ultimately the fuel requirements for the space station to accomplish its design mission.

The approach taken herein is to assume that the station is initially placed into the desired circular orbit, and the orbit is allowed to decay to a specified altitude. A Hohmann transfer maneuver is then used to return the station to a circular orbit at its original altitude. The number of such maneuvers required, for a given allowable decay altitude, is determined on a yearly basis and the propellant consumption per year required to maintain a useful orbit is also determined. Working curves yielding several variables of interest are presented for values of the ballistic coefficient equal to unity, and relations are given which allow direct application of the given results to any other value of ballistic coefficient.

Since there is apparently still some concern over the best atmospheric model to use, decay rates, decay times, and propellant consumption charts are presented for two atmospheric models - the ARDC 1959 model atmosphere (ref. 2) and the 1962 U.S. standard atmosphere (ref. 3).

The results obtained herein are found to be in general agreement with those presented by Bruce. (See ref. 4.) In his paper, Bruce obtained qualitative analytic results derived by assuming an exponential atmosphere model to determine the altitude decay time, and then used these results to obtain the propellant consumption. The results contained herein should be more accurate than those presented by Bruce, because in this analysis the actual tabulated density function is used, and the basic equation is integrated numerically.

SYMBOLS

A	characteristic area, sq ft
a	semimajor axis of orbit, ft
B	ballistic coefficient, $\frac{C_D A}{2m}$, $\frac{ft^2}{slug}$
C_D	coefficient of drag
D	drag, lb
e	eccentricity of orbit
F	thrust, lb
f	true anomaly of orbit, deg or radian
g	acceleration due to earth's gravity at sea level, 32.174, ft/sec^2
h	altitude from spherical earth, International nautical mile
I_{sp}	specific impulse of fuel, sec

m	mass, slugs
N	normal acceleration component, ft/sec^2
N_c	number of velocity corrections per year to maintain orbit between h_2 and h_1
n	mean angular velocity, radians/sec
P	period of rotation, sec
R	radial acceleration component, ft/sec^2
R_e	earth radius, 20.926×10^6 ft
r	magnitude of radius vector, ft
S	transverse acceleration component, ft/sec^2
T	tangential acceleration component, ft/sec^2
t	time, sec, or days, as noted
t_D	time to decay from h_2 to h_1, day
t_s	desired duration of orbit, sec or days
v	linear velocity of vehicle, ft/sec
Δv	corrective velocity, velocity to regain original altitude, ft/sec
Δv_t	corrective velocity needed per year, ft/sec
W_o	original weight of space station plus propellant, lb
W_p	propellant weight, lb
Z	ratio of altitude above spherical earth to earth radius, h/R_e
ρ	density of atmosphere, slugs/ft^3
μ	earth's gravitational constant, 1.408×10^{16}, ft^3/sec^2

Subscripts:

1	refers to final altitude
2	refers to initial altitude

ANALYSIS

Derivation of Basic Equation

Briefly, the approach used herein is the following: The altitude decay rate is integrated numerically to determine the decay time from the original orbital altitude to some specified altitude. The Hohmann transfer velocity to return the vehicle to its original circular orbit is determined. The number of corrective maneuvers required per year is found and combined with the Hohmann velocity to give the yearly velocity requirement to maintain the orbit. This velocity is used with the ideal velocity equation to give the propellant mass fraction required per year to maintain the orbit.

An expression for the instantaneous altitude decay rate can easily be obtained directly from Lagrange's planetary equations. (See, for example, ref. 5.) The one equation needed here is

$$\frac{da}{dt} = \frac{2}{nr\sqrt{1 - e^2}}\left[\left(er \sin f\right)R + a\left(1 - e^2\right)S\right] \tag{1}$$

where the radial and transverse acceleration components, R and S, can be expressed in terms of tangential and normal acceleration component, T and N, as

$$R = \frac{e \sin f}{\sqrt{1 + e^2 + 2e \cos f}} T - \frac{1 + e \cos f}{\sqrt{1 + e^2 + 2e \cos f}} N \tag{2}$$

$$S = \frac{1 + e \cos f}{\sqrt{1 + e^2 + 2e \cos f}} T + \frac{e \sin f}{\sqrt{1 + e^2 + 2e \cos f}} N \tag{3}$$

For the circular-orbit drag decay problem,

$$a = r$$

$$e = 0$$

$$T = -\frac{D}{m}$$

$$N = 0$$

and hence

$$R = 0$$

$$S = -\frac{D}{m}$$

Equation (1) becomes

$$\frac{dr}{dt} = -\frac{2}{n}\frac{D}{m} \tag{4}$$

The mean angular velocity n can be expressed as

$$n = \frac{2\pi}{P} = \frac{\sqrt{\mu r}}{r^2} \tag{5}$$

and with

$$\frac{D}{m} = \frac{1}{2}\rho v^2 \frac{C_D A}{m} = \left(\frac{C_D A}{2m}\right)\rho \frac{\mu}{r} \tag{6}$$

where

$$v^2 = \frac{\mu}{r}$$

equation (4) can be written

$$\frac{dh}{dt} = \frac{dr}{dt} = -2B\rho\sqrt{\mu r} \tag{7}$$

with the definition

$$B = \frac{C_D A}{2m} \tag{8}$$

Equation (7) is the basic equation used herein and forms the starting point for the analysis.

Note that equation (7) is linear in B, which is herein assumed to be constant. Thus, a value of $B = 1$ is used herein, and this proportionality may be amended so as to use the charts presented for any other value of B.

Atmospheric and Earth Models

Because of the wide spread in some of the experimental data available on the density of the atmosphere at the higher altitude (>60 nautical miles), and because of the known fluctuation in the density with latitude, night and day, and sunspot activity, there is apparently still some indecision as to the best standard atmospheric model to use. For this reason, the results in this paper are presented for two atmospheric models, the 1959 ARDC model atmosphere (ref. 2) and the 1962 U.S. standard atmosphere (ref. 3). The density as a function of altitude is shown in figure 1 for both model atmospheres.

The other physical constants used in this paper are 20.926×10^6 feet for the radius of the earth, and 1.408×10^{16} cubic feet per second^2 for the earth's gravitational constant μ. The nautical mile referred to throughout the text is the International nautical mile, 6076.11549 feet.

Decay Times and Propellant Mass Fraction

The decay rates from equation (7), using both atmospheric models are shown in figure 2, and form the basic data for the analysis to follow. The time to decay from any initial altitude h_2 to some final altitude h_1 is then given by numerically integrating equation (7) as

$$t_D = -\int_{h_2}^{h_1} \frac{dt}{dh}\,dh \tag{9}$$

Three basic assumptions are made at this point:

(1) The ballistic parameter B is constant.

(2) The earth is not rotating (stationary atmosphere).

(3) As the orbit decays, it remains in a succession of circular orbits (that is, the velocity at each altitude is equal to the local circular velocity).

The results of this integration are shown in figure 3 for the 1962 U.S. standard atmosphere and in figure 4 for the 1959 ARDC model atmosphere. In these figures, the initial altitude h_2 has been varied from 300 to 160 nautical miles. The decay times in days are then plotted against the final altitude h_1, for a value of ballistic coefficient B equal to unity. For a general value of B not equal to unity, the decay time can be found from

$$t_{D,B} = \frac{1}{B}\,t_{D,B=1} \tag{10}$$

where $t_{D,B=1}$ is taken directly from figure 3 or 4.

Also shown in figures 3 and 4 are lines of constant decay time. For example, reference to figure 3(a) shows that the dashed lines marked 95 percent, 90 percent, and 50 percent intersect the $h_2 = 300$ nautical miles altitude curve at 198, 216, and 273 nautical miles, respectively. This condition means that a satellite with $B = 1$, initially placed into a circular orbit at 300 nautical miles, spends 95 percent of its total lifetime above 198 nautical miles, 90 percent of its lifetime above 216 nautical miles, and 50 percent of its lifetime above 273 nautical miles. Total lifetime is here defined as the time to decay from the initial altitude h_2 down to $h_1 = 100$ nautical miles, since these calculations indicate that a satellite with $B = 1$ in a 100-nautical-mile circular orbit would remain in orbit for something less than 1 or 2 days.

These decay data are conveniently summarized in table I which gives the decay times in days over 5-nautical-mile intervals, starting at 300 nautical miles for both atmosphere models.

If the decay times, taken from figure 3 or 4, are divided into 365, the number of days in a year, the number of corrective maneuvers which must be made per year to maintain the spacecraft orbit between the altitude limits of h_2 and h_1 can be found.

$$N_c = \frac{365}{t_D} \tag{11}$$

For a value of $B \neq 1$,

$$N_c = B\left(N_c\right)_{B=1}$$

The number of such corrections per year is shown in figure 5, again for both atmospheric models. Note that, as one could expect, the number of corrections per year increases as $h_2 - h_1$ decreases and approaches infinity as $h_2 - h_1$ approaches zero. This is the case when thrust is applied continuously with the thrust force equal in magnitude to the drag force.

In order to maintain a useful orbit, it was assumed that the spacecraft initially placed into a circular orbit at altitude h_2 was allowed to decay to a circular orbit at altitude h_1. A Hohmann transfer maneuver was then used to return the station to its original orbital condition. An expression for the total velocity required to perform this corrective maneuver for any two altitude limits h_2 and h_1 has been derived by Thomson (ref. 6) and is shown below, where $r_1 = R_e + h_1$ and $r_2 = R_e + h_2$:

$$\Delta v = \sqrt{\frac{\mu}{r_1}}\left[\sqrt{\frac{2\left(r_2/r_1\right)}{1 + \left(r_2/r_1\right)}}\left(1 - \frac{r_2}{r_1}\right) + \sqrt{\frac{r_1}{r_2}} - 1\right] \tag{12}$$

Equation (12) is displayed graphically in figure 6 for the altitude range of interest here. It can be remarked that for $h \ll R_e$, equation (12) can be simplified considerably. If $Z = h/R_e$, an expression correct to the second order in Z that can be used in place of equation (12) is

$$\Delta v = \sqrt{\frac{\mu}{r_1}}\left[\frac{Z_2 - Z_1}{2} + \frac{(Z_2 - Z_1)^2}{4} - \frac{\left(Z_2^2 - Z_1 Z_2\right)}{2}\right] \tag{13}$$

The first term in square brackets is the first-order approximation and can be used if the altitude difference $h_2 - h_1$ does not exceed 10 or 20 nautical miles. Equation (13) is presented here as a convenience. This approximation to equation (12) was not used in the calculations presented herein.

The total velocity required per year for orbital maintenance can be found by multiplying the velocity requirement per corrective maneuver from figure 6, by the number of corrections per year which must be made:

$$\Delta v_t = N_c \Delta v \tag{14}$$

For $B \neq 1$,

$$\Delta v_t = B\left(\Delta v_t\right)_{B=1}$$

This total maintenance velocity is shown plotted in figure 7. Here it should be noted that, as $h_2 - h_1$ approaches zero, the maintenance velocity remains finite; thus, as will be shown subsequently, a finite amount of propellant is required for orbital maintenance even in the case where thrust is applied continuously.

It might be remarked that in some instances the natural decay time between two altitude limits exceeds 1 year. For these cases, the total velocity increment shown in figure 7 should be taken as average yearly rates. For example, using the 1962 U.S. standard atmosphere, figure 3(a) shows that a correction must be applied every 1,630 days if the orbit is allowed to decay from 300 to 100 nautical miles. This is every 4.47 years. From figure 6, the correction velocity required for this maneuver is 693 feet per second, or an average velocity penalty of 153 feet per second, as shown in figure 7(a).

If it is assumed that no drag acts over the comparatively short time that the corrective maneuvers take place, it is possible to find the propellant mass fraction required to maintain the orbit from the ideal velocity equation:

$$\Delta v_t = -gI_{sp} \log_e\left(1 - \frac{W_p}{W_o}\right) \tag{15}$$

Equation (15) is used herein. However, if $\Delta v_t \ll gI_{sp}$, equation (15) can be simplified to

$$\frac{W_p}{W_o} \approx \frac{\Delta v_t}{gI_{sp}} \tag{16}$$

The differences in calculating W_p/W_o from equations (15) and (16) begin to be noticeable when Δv_t is of the order of 3 percent of gI_{sp}, and hence equation (16) is not used herein. The only difficulty encountered here is that the final results for propellant mass fraction required for orbital maintainence could not be generalized for all values of specific impulse I_{sp}. A value of $I_{sp} = 300$ seconds was used here. For any other value of I_{sp}, the conversion formula

$$\frac{W_p}{W_o} = 1 - \left[1 - \left(\frac{W_p}{W_o}\right)_{\substack{I_{sp}=300 \\ B=1}}\right]^{\frac{300B}{I_{sp}}} \tag{17}$$

must be used. Note that the equation also makes the correction for B simultaneously with the correction for I_{sp}.

The propellant mass fraction per year required to maintain the orbit between the limits of h_2 and h_1 was calculated by using equation (15), and the results are given in figure 8. The dashed lines indicate the propellant mass fraction required if the orbit is allowed to decay 10 percent or 50 percent of the initial altitude. The line marked continuous thrust is the propellant mass fraction required if the orbit is not allowed to decay at all, but is maintained by a continuously thrusting rocket motor whose thrust magnitude F is equal to the decay force experienced by the satellite. Thus, with F = D,

$$\frac{W_p}{W_o} = B\rho \frac{t_s}{I_{sp}}\left(\frac{\mu}{gr}\right) \tag{18}$$

where t_s is the desired duration of orbital sustenance.

In figure 9 is shown the propellant mass fraction required if the orbit is allowed to decay the indicated fraction of its original altitude. This figure was obtained from figure 8.

RESULTS AND DISCUSSION

Working charts have been presented which permit the simple determination of several parameters which are useful in determining the propellant requirements to sustain an orbiting spacecraft in a useful orbit between two limiting altitudes in the range 160 to 300 nautical miles. These parameters are:

(1) The decay time, the time it takes the satellite to decay from a circular orbit at altitude h_2 to a circular orbit at altitude h_1 when acted upon solely by aerodynamic drag (see figs. 3 and 4)

(2) The number of Hohmann-type correction maneuvers required per year to transfer the spacecraft from a circular orbit at altitude h_1 to a circular orbit at altitude h_2 (see fig. 6)

(3) The total velocity penalty per year associated with these correction maneuvers (see fig. 7)

(4) The propellant mass fraction required per year to perform these correction maneuvers (see figs. 8 and 9)

It might be noted, incidentally, that the effective lifetime of a spacecraft originally in a circular orbit at altitudes ranging from 160 to 300 nautical miles can also be found from figures 3 and 4, with $h_1 = 100$ nautical miles.

All the data presented have been valid only for a value of ballistic coefficient $B = 1$. For any other value of B, the following conversion formulas can be used:

$$t_D = \frac{(t_D)_{B=1}}{B}$$

$$N_c = (N_c)_{B=1} B$$

$$\Delta v_t = (\Delta v_t)_{B=1} B$$

The conversion formula for W_p/W_o for $I_{sp} \neq 300$ seconds and $B \neq 1$ is given by equation (17).

One possible application of the results presented herein will be given here. Because of the recently discovered manmade radiation belt off the east coast of Brazil, it might be necessary to place permanent manned orbiting spacecraft in orbits of lower altitudes than desired from a lifetime point of view because of the excessive shielding requirements necessary to protect the occupants of the station. In figure 10 is presented the shielding weight in pounds per square foot of a CH_2-type plastic necessary to keep the radiation to which the crew is assumed to be exposed down to presently acceptable limits. (See, for example, references 7 and 8 for calculation procedures and tolerance limits.) It will be noticed that the shielding requirements increase with increasing orbital altitude. This shielding requirement represents a reduced useful payload in orbit, since it is required in addition to the basic structural weight. Since the propellant requirements for orbital sustenance decrease with increasing altitude it seems likely that the sum of shielding weight and propellant weight might have a minimum when plotted against orbital altitude. The altitude at which this minimum occurs, then, represents an optimum altitude for orbiting if the only consideration is useful payload in orbit and if the only weight penalties considered are those associated with shielding requirements and orbital-sustenance propellant requirements. The example of this calculation is shown in figure 11, in which it is assumed that a 50,000-pound spacecraft with $B = 1$, is to maintain a circular orbit with continuous thrust, and is required to shield 1,000 square feet of housing area for the crew. For this spacecraft, it is seen that an orbital altitude of about 226 nautical miles minimized the total shielding weight and propellant weight for 1 year.

It can be noted from figure 8 or 9 that, as pointed out by Bruce (ref. 4), the propellant requirement is minimized if one uses continuous thrust to maintain the orbital altitude. However, except in the case when a system such as an ion rocket or solar sail can be used, the practicability of such a system is questionable because of the extremely low thrust magnitudes required. Thus, the scheme of allowing the orbit to decay and then apply a correction maneuver seems quite feasible with conventional rocket motors of reasonable size, since the correction velocity requirements per maneuver (fig. 5) are not excessively large.

CONCLUDING REMARKS

A set of working charts is presented which allows the rapid determination of the propellant requirements for maintaining an earth orbital vehicle in a useful orbit between the altitude limits of 160 to 300 nautical miles. The following assumptions were made in the analysis:

(1) The earth is spherical and nonrotating.

(2) As the orbit decays, the velocity of the satellite is always equal to the local circular velocity.

(3) The ballistic coefficient B is constant.

(4) The drag acting on the vehicle during the transfer maneuver is negligible.

Two atmospheric models were used, the 1959 ARDC model atmosphere, and the 1962 U.S. standard atmosphere.

It was found that the minimum propellant requirement occurs when the satellite is not allowed to decay at all but is maintained by a continuously thrusting rocket motor, whose thrust is equal in magnitude to the decay force experienced by the spacecraft. This condition has been pointed out previously by Bruce.

At the higher altitudes, above about 260 nautical miles, very modest amounts of propellant, of the order of 1 percent of the initial weight, are required per year if the orbit is not allowed to decay a large percentage of the original orbit altitude. In the lower altitude region, below about 180 nautical miles, the yearly propellant consumption amounts to something of the order of 10 percent of the total spacecraft weight, or greater. This large propellent requirement, however, does not make orbital sustenance at these altitudes impractical because rendezvous techniques, which allow maintenance fuel to be brought to the spacecraft at periodic intervals, can significantly reduce the propellant mass fraction required during the period between two successive rendezvous missions.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., April 12, 1963.

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TABLE I.- DECAY TIMES IN DAYS OVER 5-NAUTICAL-MILE INTERVALS

FROM 300 NAUTICAL MILES TO 100 NAUTICAL MILES

FOR TWO ATMOSPHERIC MODELS. B = 1.0

Interval, nautical miles	Decay times, days for -	
	1962 U.S. standard atmosphere	1959 ARDC atmosphere
300 to 295	194.0	137.9
295 to 290	173.1	123.9
290 to 285	153.9	110.9
285 to 280	136.1	98.5
280 to 275	120.5	87.1
275 to 270	106.7	77.2
270 to 265	94.2	68.1
265 to 260	83.3	60.0
260 to 255	73.8	53.0
255 to 250	65.6	47.0
250 to 245	58.2	41.7
245 to 240	52.8	36.8
240 to 235	46.2	32.4
235 to 230	38.7	27.3
230 to 225	33.4	24.6
225 to 220	29.0	21.3
220 to 215	25.3	18.5
215 to 210	22.0	16.0
210 to 205	19.0	13.7
205 to 200	16.5	12.0
200 to 195	14.39	10.60
195 to 190	12.18	9.18
190 to 185	10.29	7.93
185 to 180	8.70	6.83
180 to 175	7.33	5.81
175 to 170	6.18	4.91
170 to 165	5.19	4.15
165 to 160	4.37	3.50
160 to 155	3.66	2.97
155 to 150	3.12	2.48
150 to 145	2.66	2.13
145 to 140	2.22	1.81
140 to 135	1.83	1.53
135 to 130	1.48	1.27
130 to 125	1.16	1.02
125 to 120	.90	.83
120 to 115	.69	.67
115 to 110	.55	.53
110 to 105	.43	.41
105 to 100	.34	.33

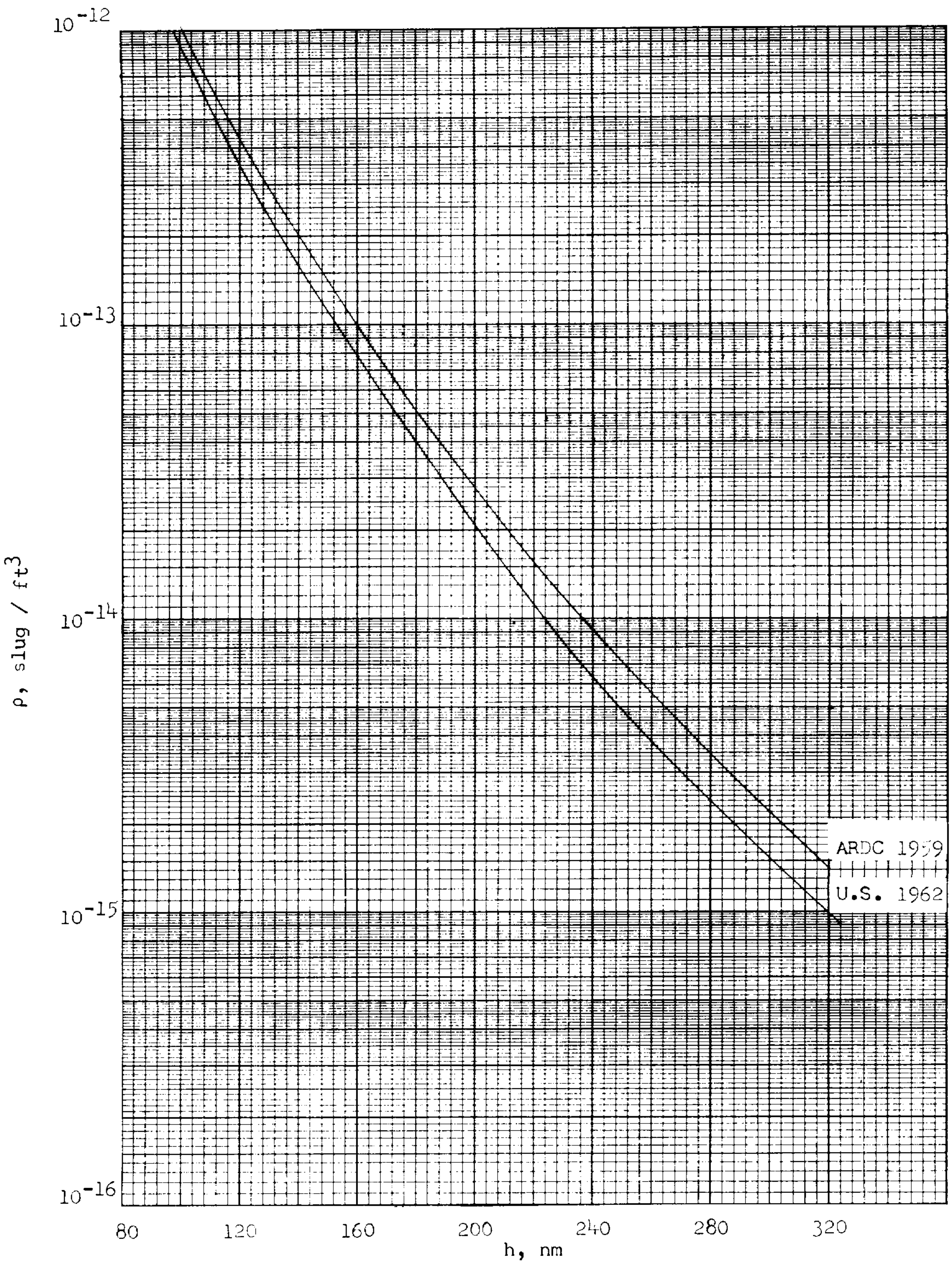


Figure 1.- Comparison of the densities of the 1959 ARDC atmosphere and the U.S. standard atmosphere 1962.

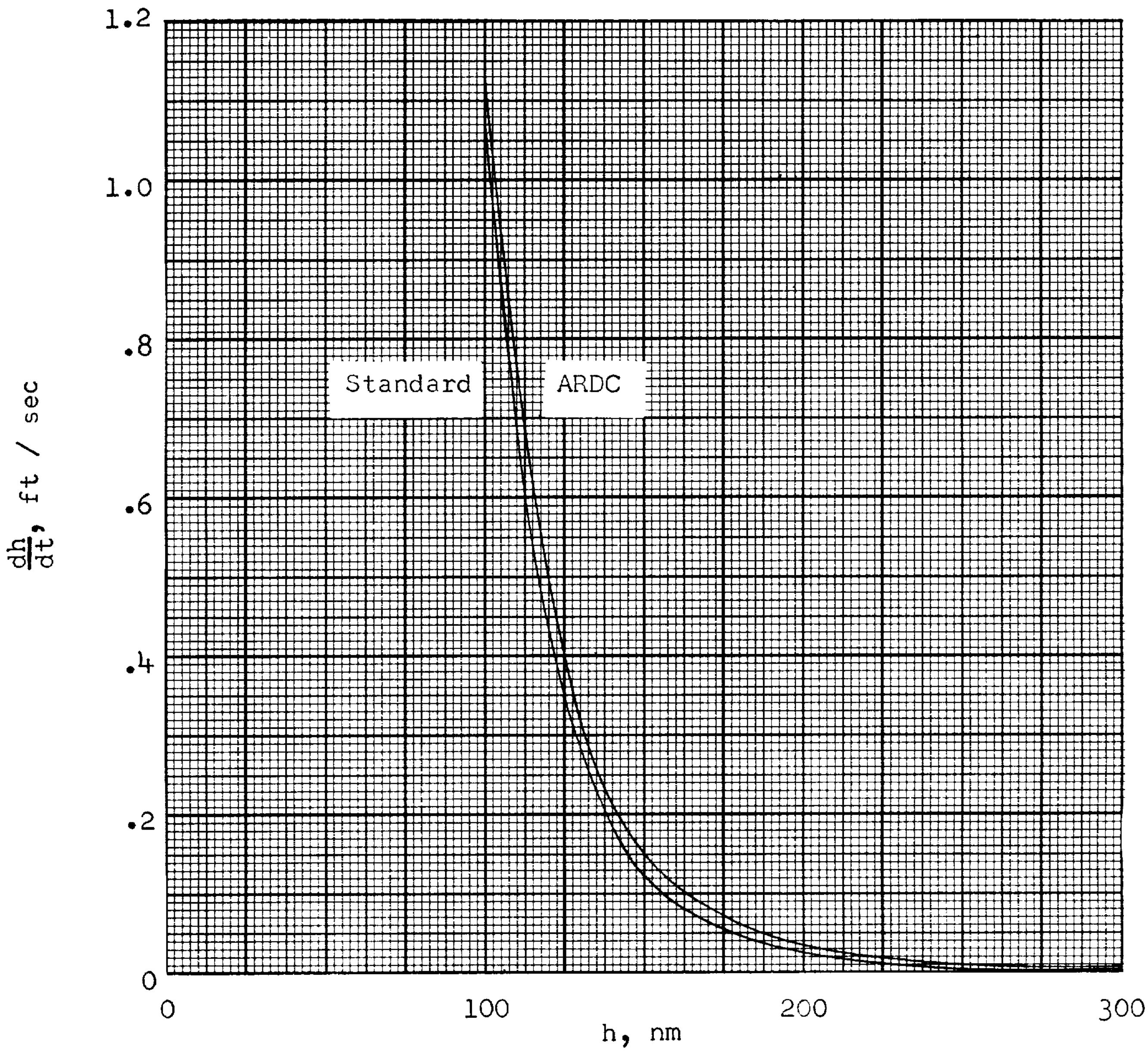
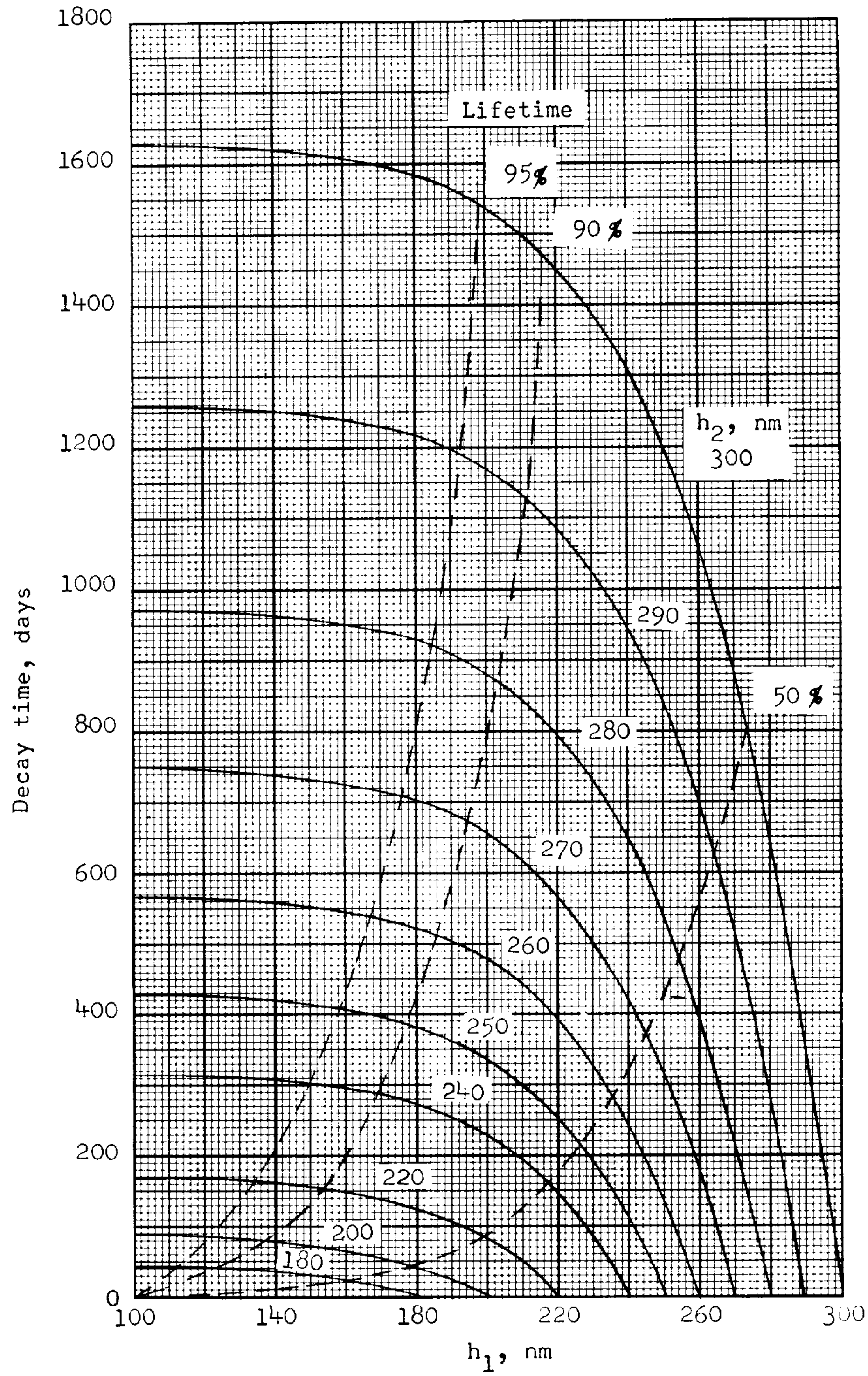
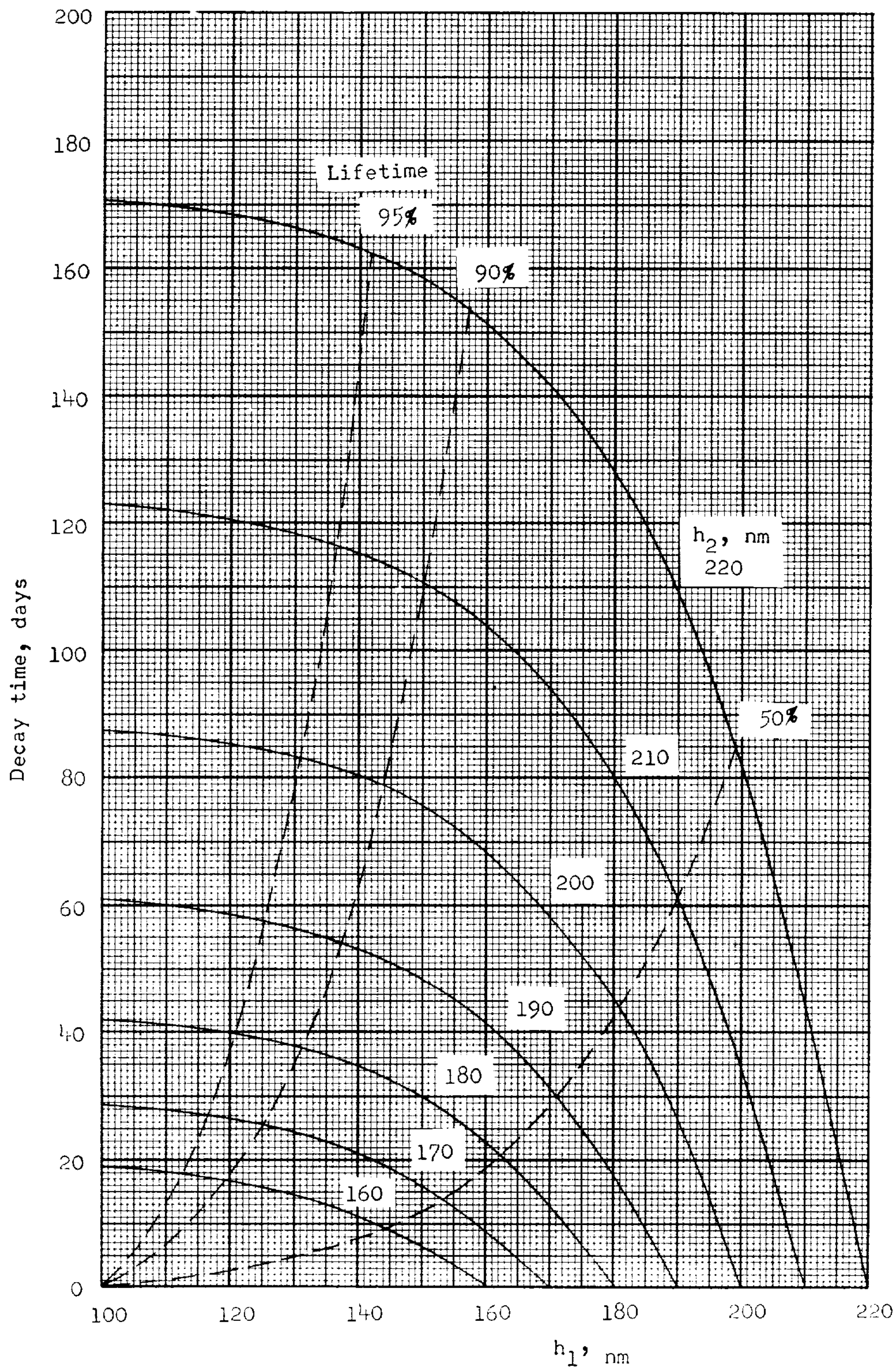


Figure 2.- Instantaneous decay rates for both atmospheric models. B = 1.



(a)

Figure 5.- Decay time between two altitudes h_2 and h_1. B = 1, U.S. standard atmosphere 1962.



(b)

Figure 3.- Concluded.

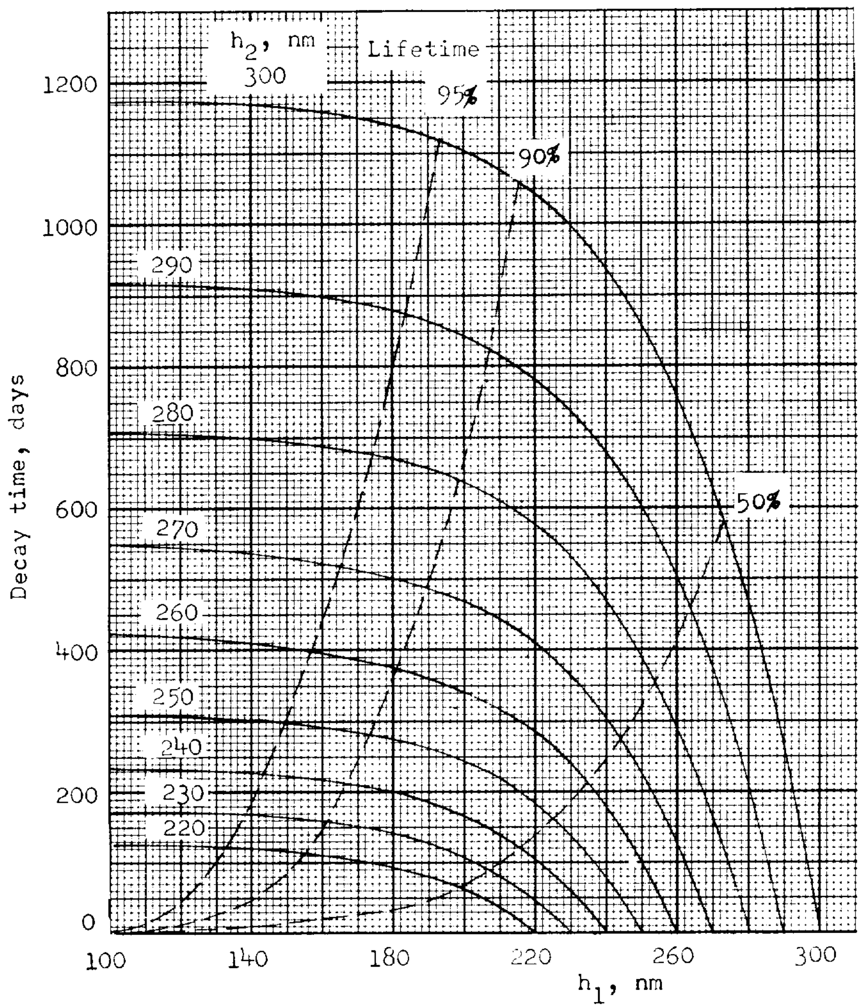


Figure 4.- Decay time between two altitudes h_2 and h_1. $B = 1$, 1959 ARDC atmosphere.

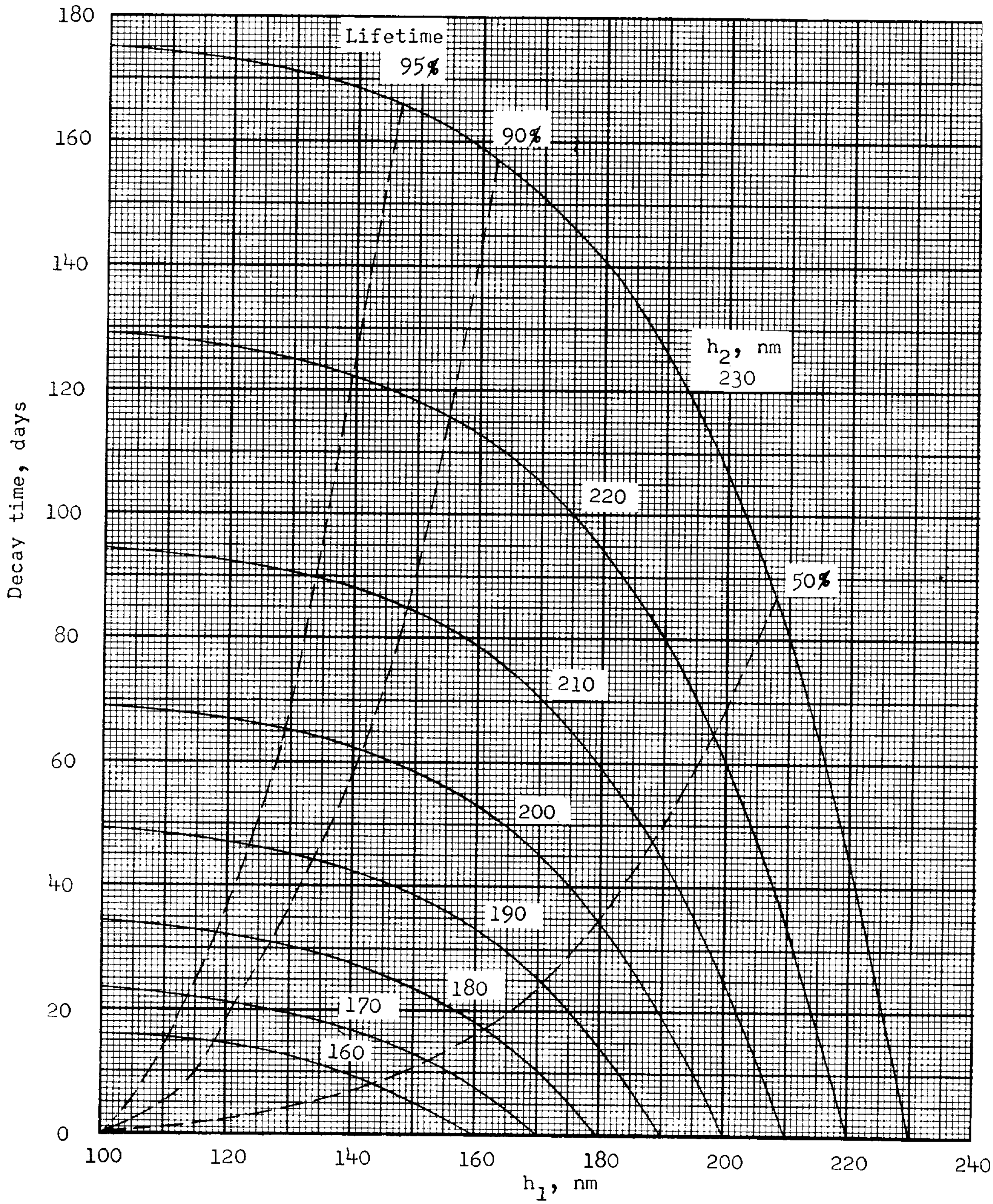
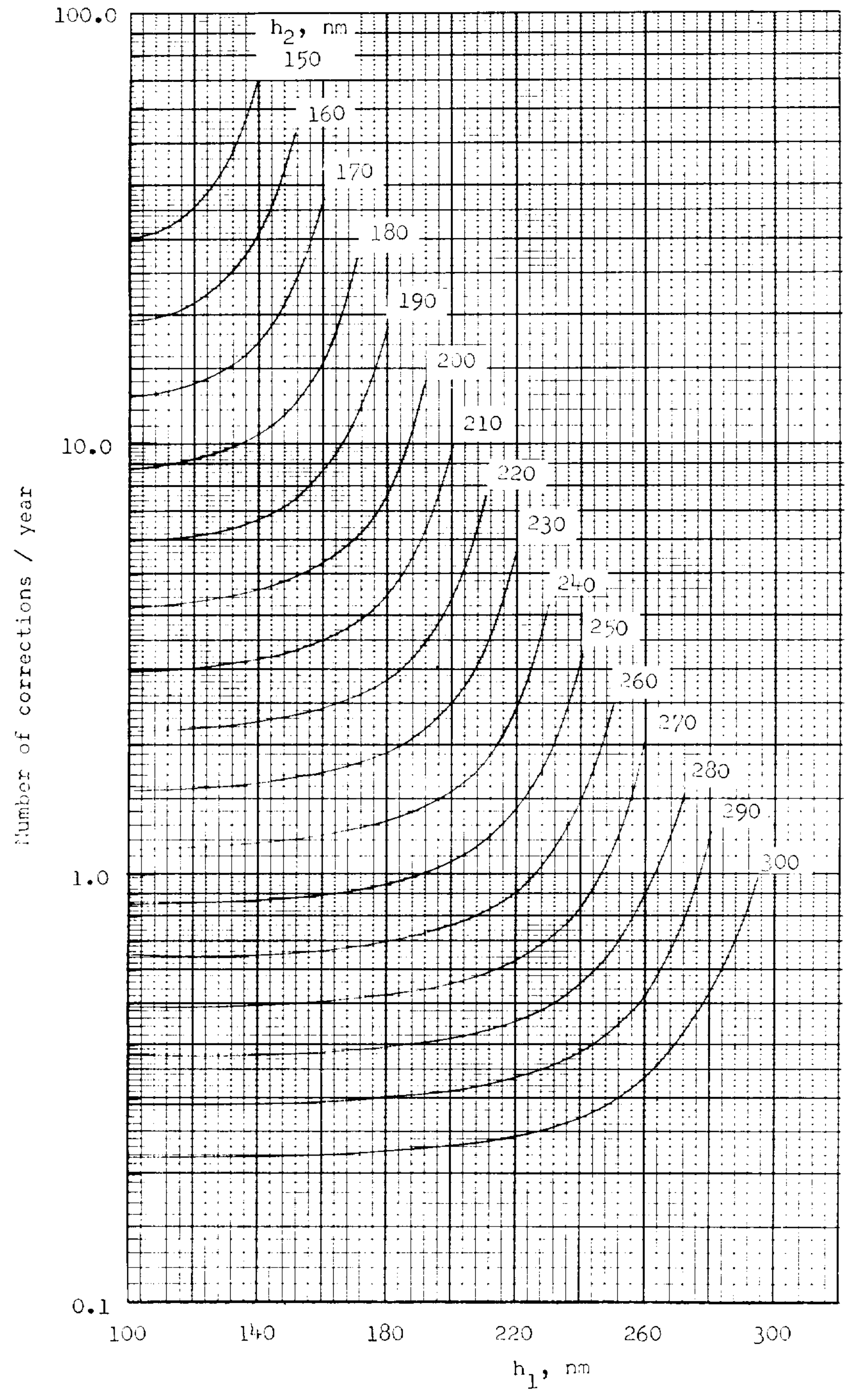
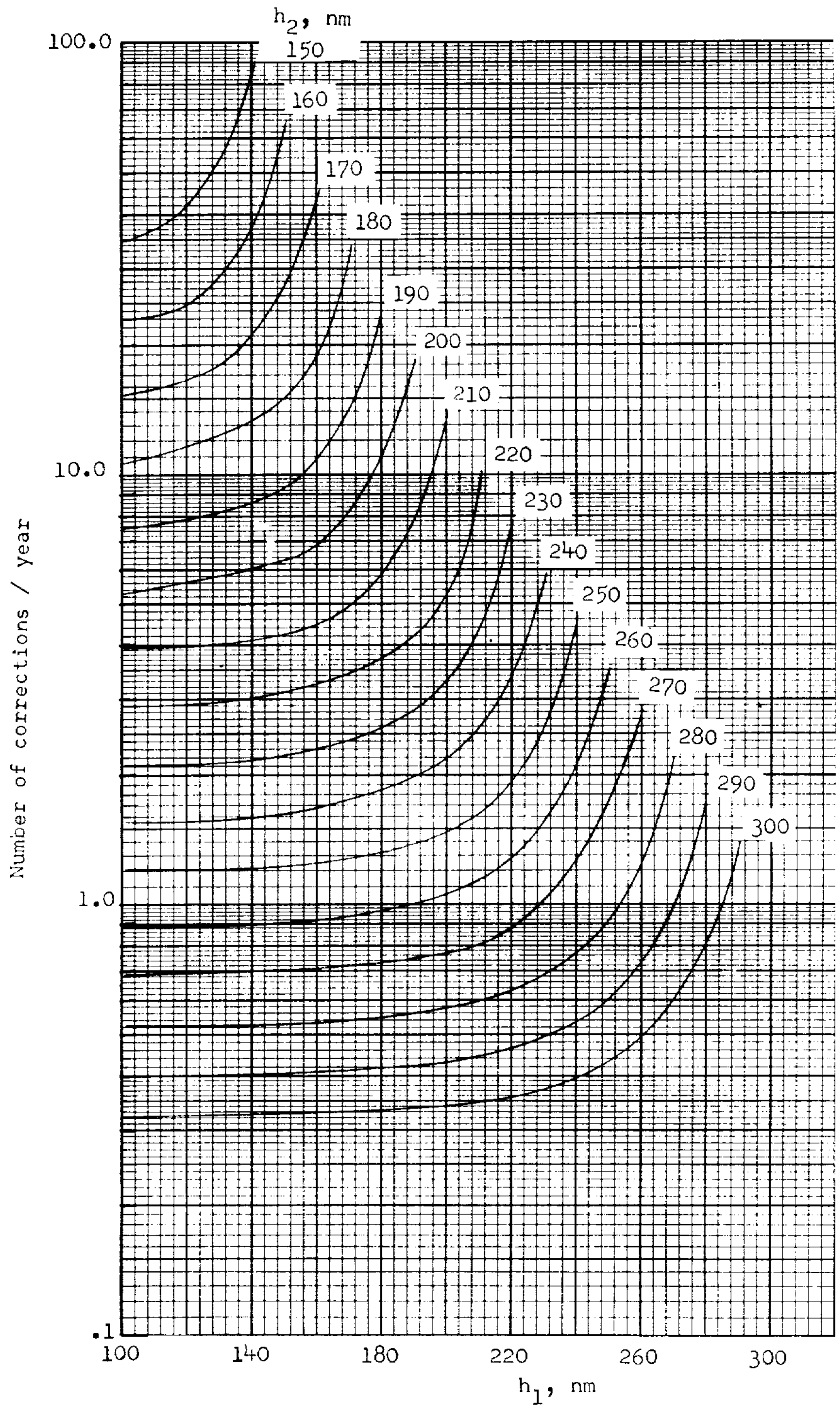


Figure 4.- Concluded.



(a) B = 1; U.S. standard atmosphere 1962.

Figure 5.- Number of Hohmann corrections required per year to maintain orbit between altitude limits of h_2 and h_1.



(b) B = 1; 1959 ARDC atmosphere.

Figure 5.- Concluded.

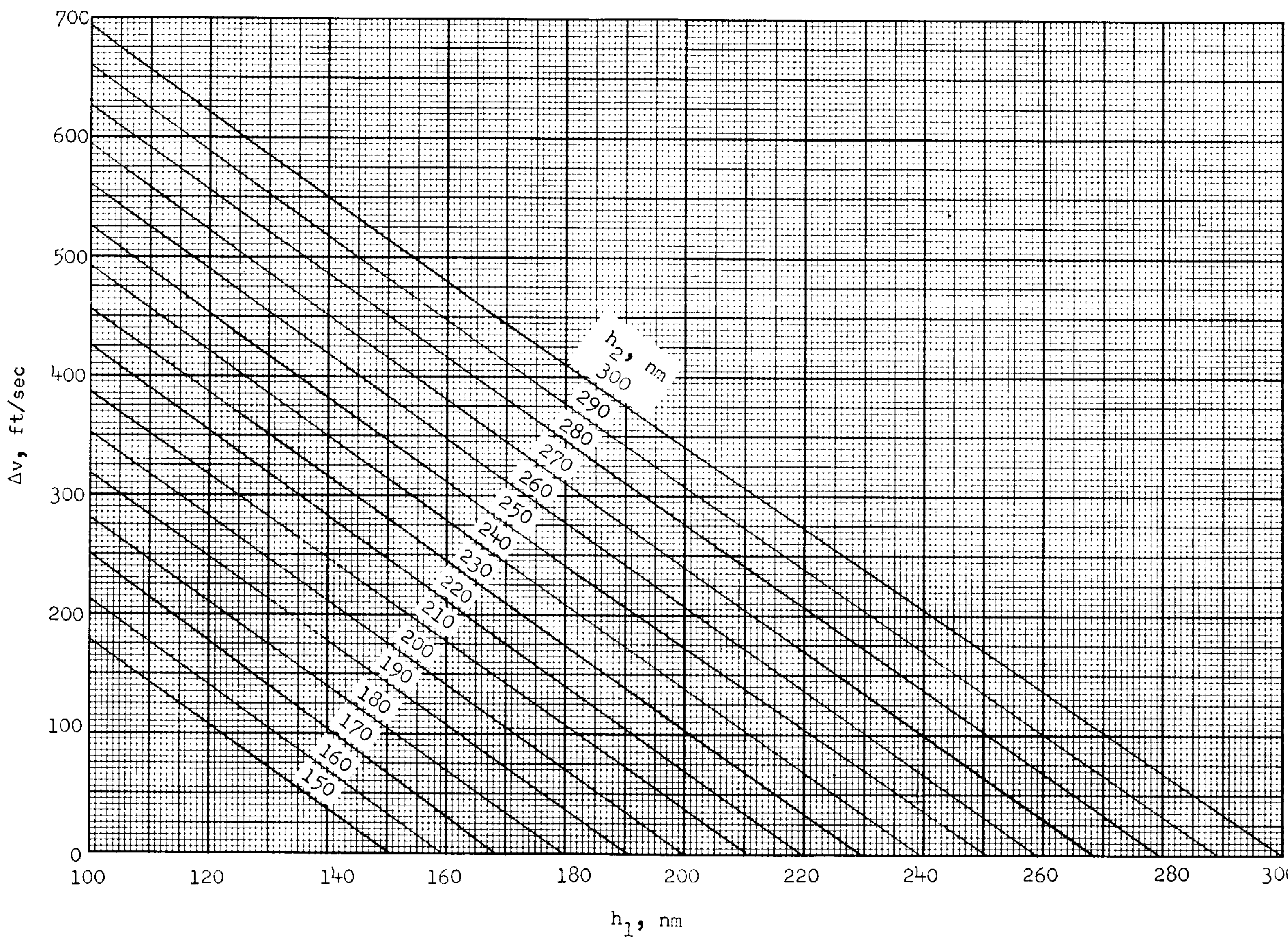
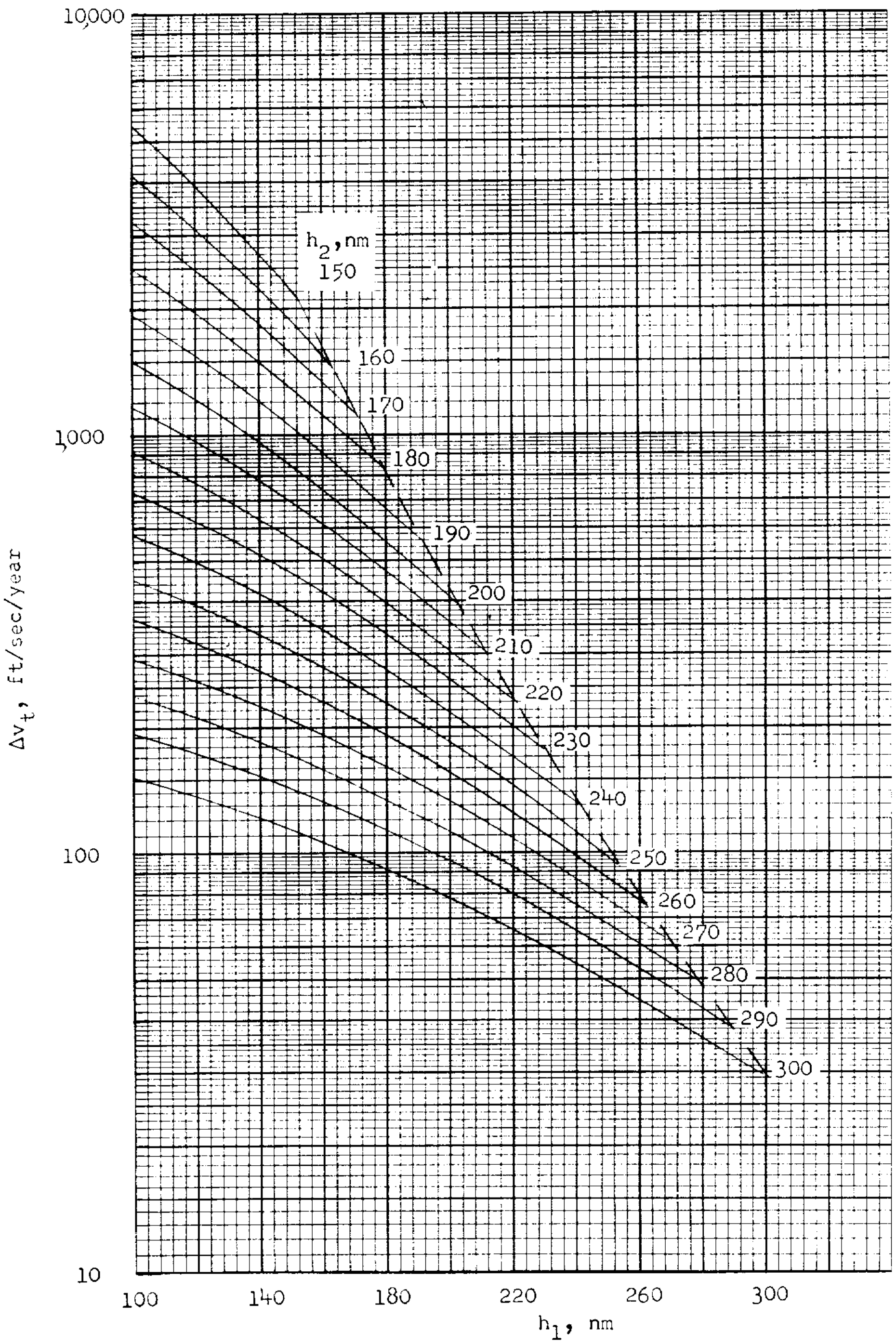
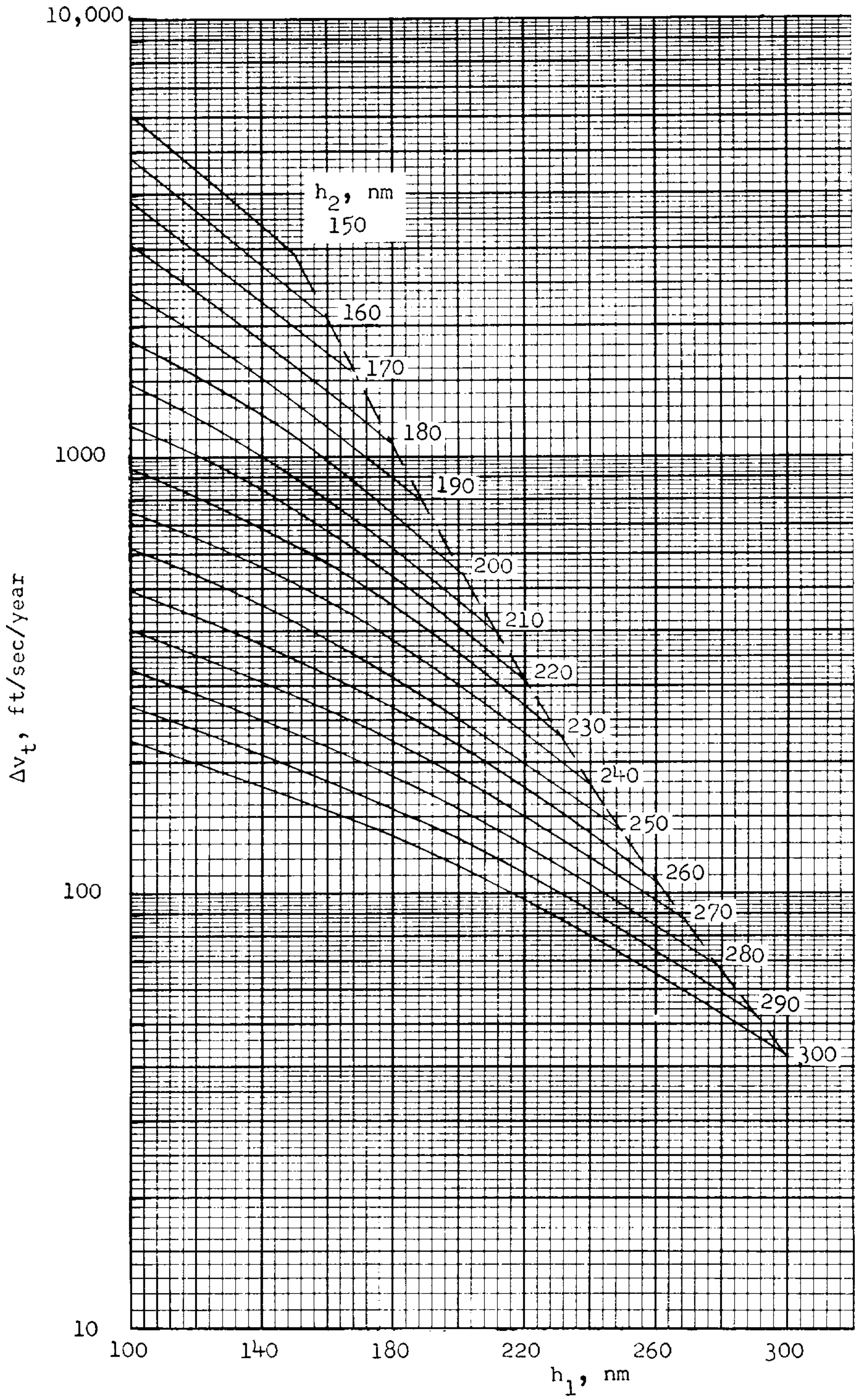


Figure 6.- Hohmann transfer velocity requirements between altitudes h_2 and h_1. Transfer from a circular orbit at h_1 to a circular orbit at h_2.



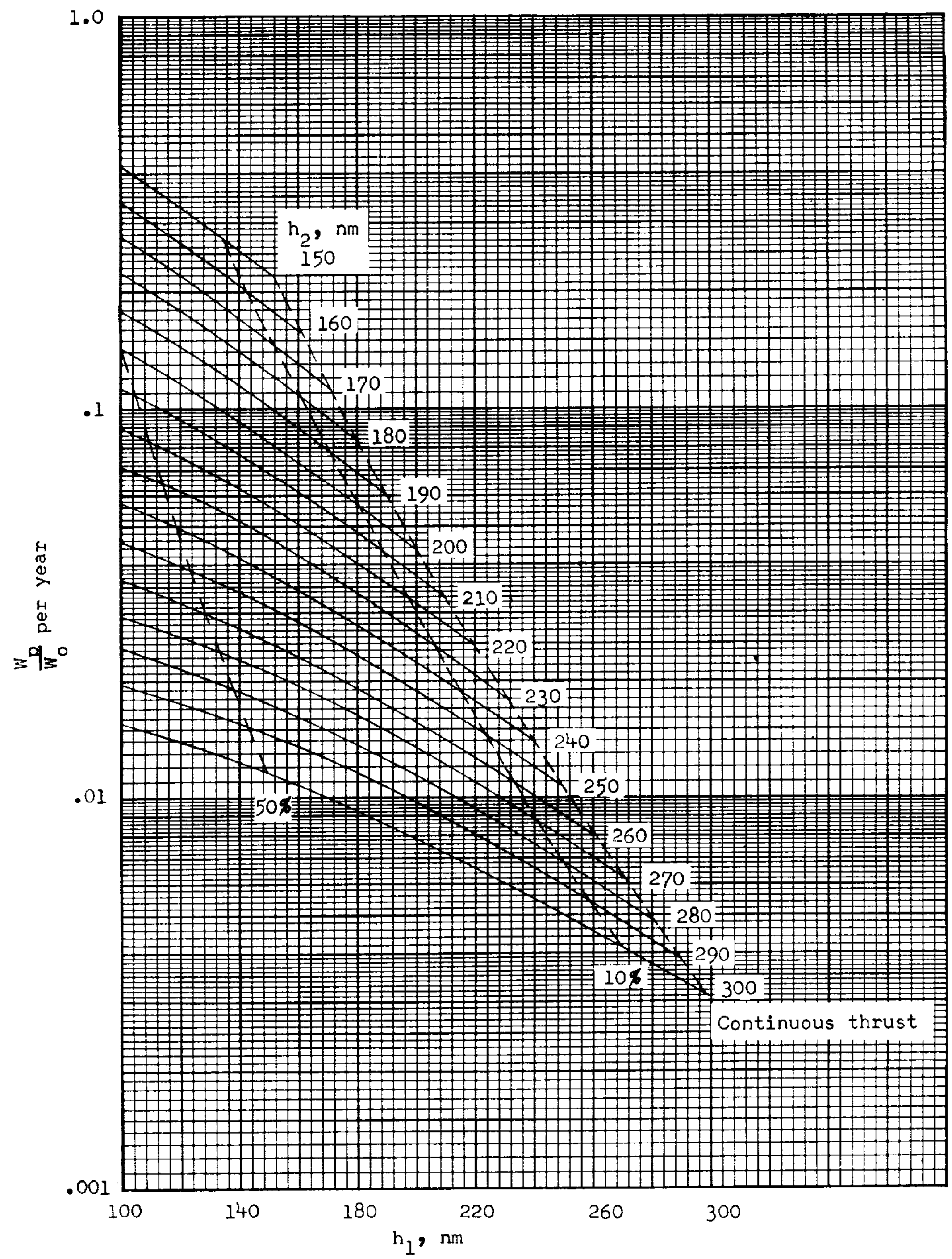
(a) B = 1; U.S. standard atmosphere 1962.

Figure 7.- Total velocity requirement per year to maintain orbit between altitude limits of h_2 and h_1.



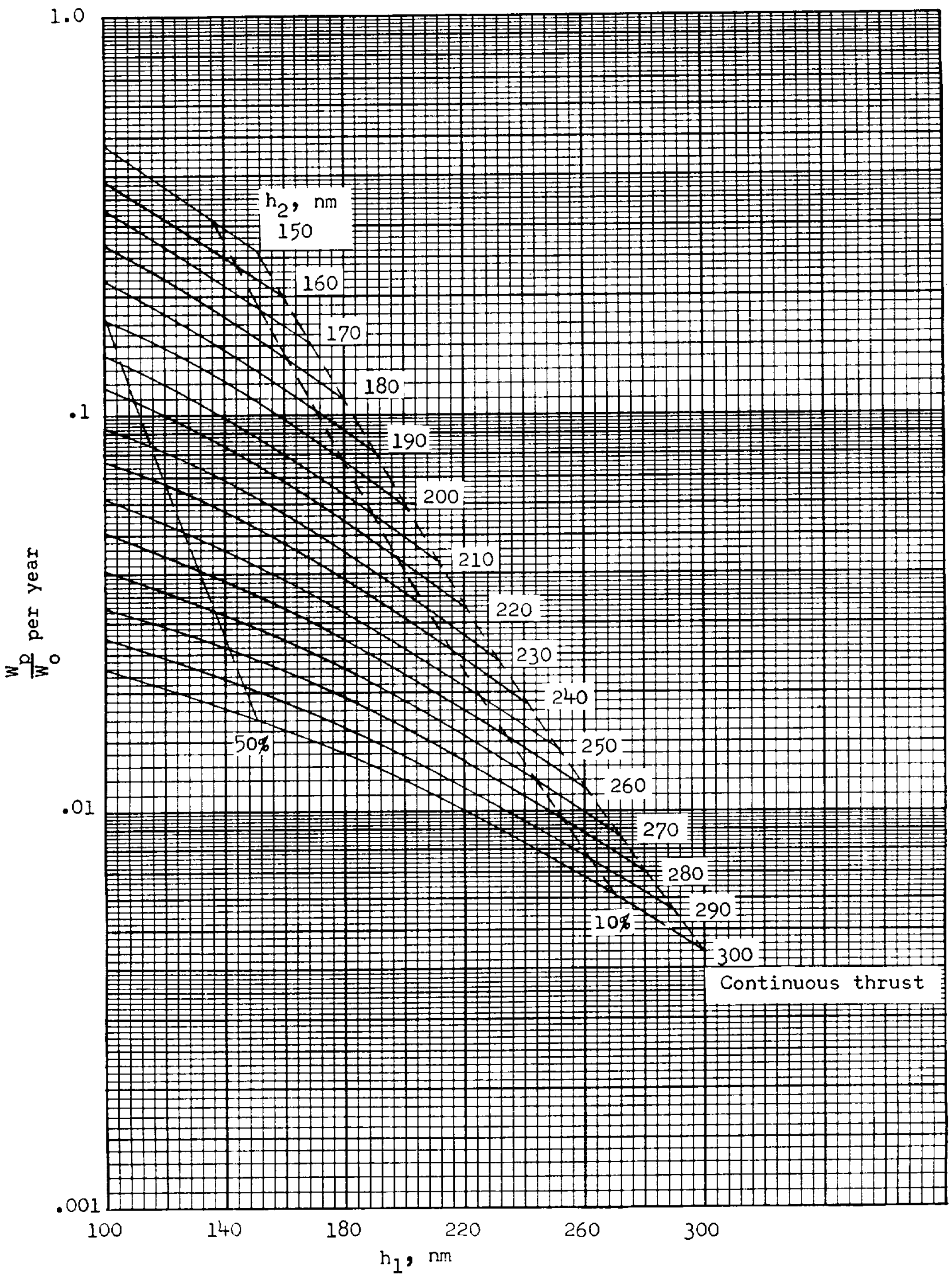
(b) B = 1; 1959 ARDC atmosphere.

Figure 7.- Concluded.



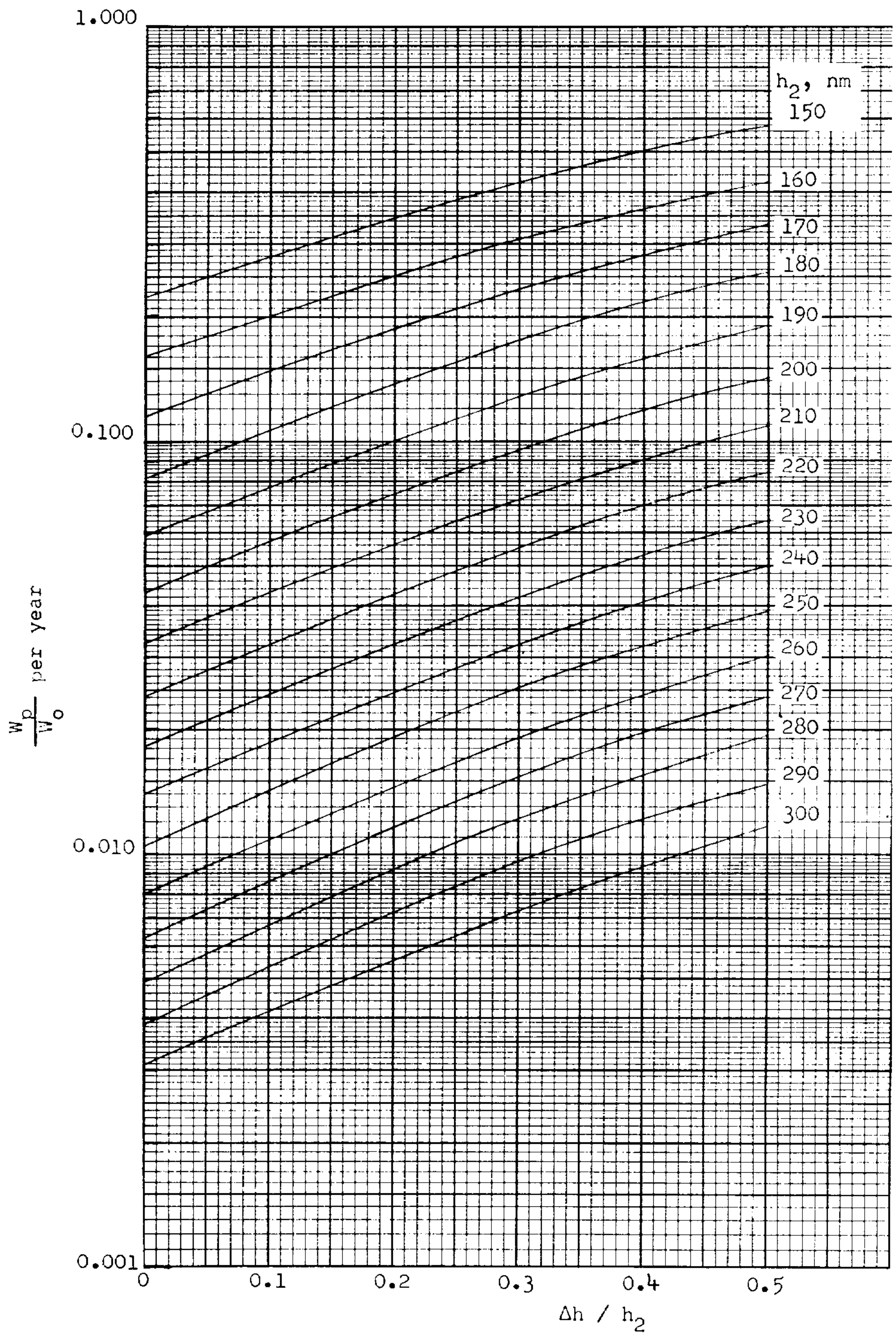
(a) B = 1; U.S. standard atmosphere 1962.

Figure 8.- Propellant mass fraction required per year to maintain orbit between altitude limits of h_2 and h_1.



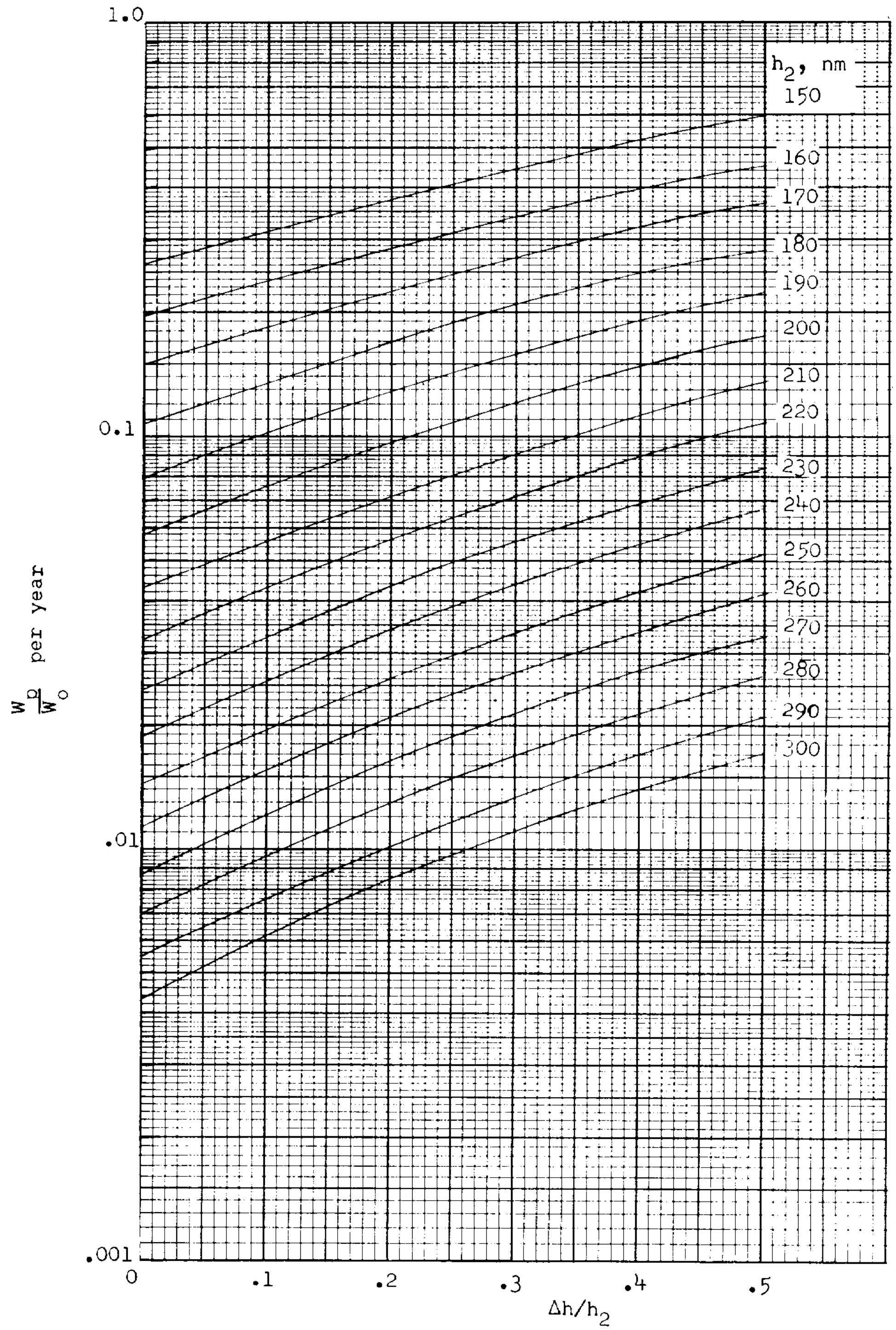
(b) B = 1; 1959 ARDC atmosphere.

Figure 8.- Concluded.



(a) B = 1; U.S. standard atmosphere 1962.

Figure 9.- Propellant mass fraction required per year to maintain orbit for an allowable decay to a given percentage of the original orbit altitude.



(b) B = 1; 1959 ARDC atmosphere.

Figure 9.- Concluded.

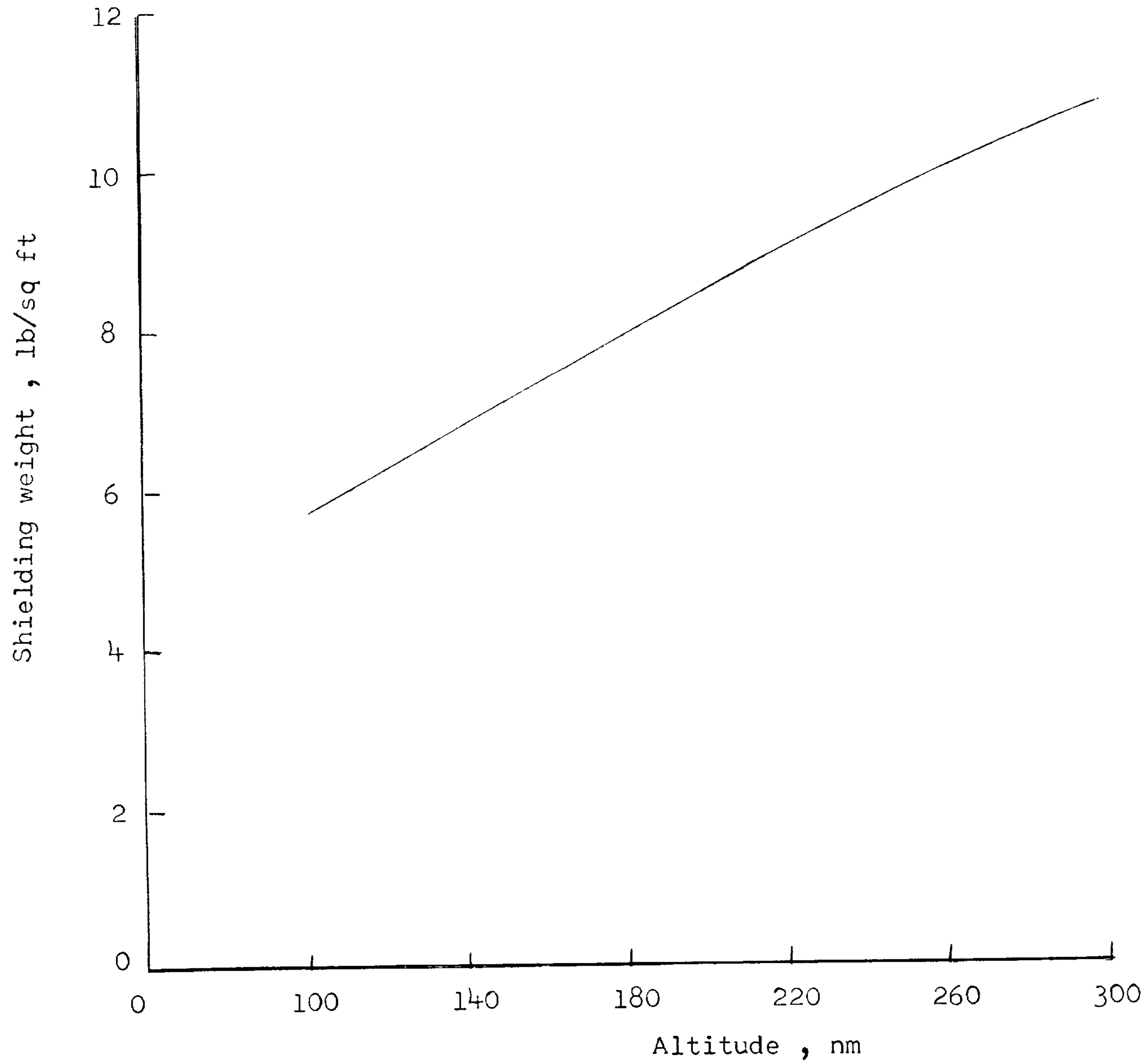


Figure 10.- Shielding weight per square foot required for a 30° inclined orbit.

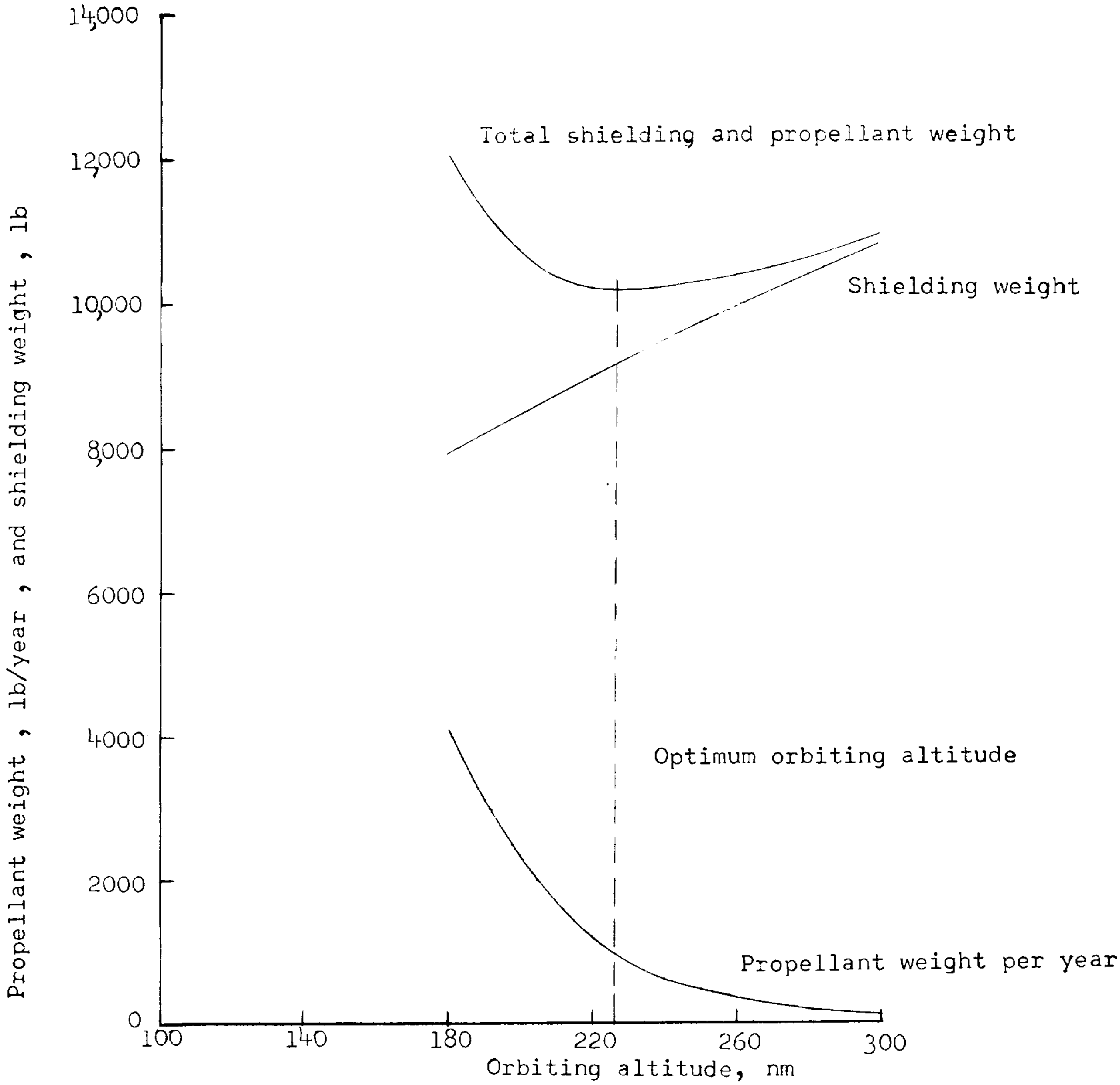


Figure 11.- Graphic determination of the optimum orbiting altitude giving consideration only to shielding and propellant weight requirements.